The Bubby Cube

One day in math class, poor Chloe watched her teacher assemble a cube from 27 smaller cubes. At home that afternoon she attempted to make the same structure from her modest set of blocks. She found that she was three cubes short. One entire row on the top level was vacant.

Chloe's classmate Bubby MacFarlane had an enormous bag of cubes. In the middle of his living room, he set about the task of building the world's largest cube ever made by a six-year-old. Instead of attaining success, he had the biggest tantrum ever when he too ended up with a vacant row on the top level. Poor Chloe comforted Bubby the next day in school.

We don't know how many blocks Bubby owned, but like Chloe's 24, we know it must be a multiple of six.

Can you show how we know this with a picture or a little algebra?

Solution

The "Bubby" shape can be reconfigured so that the number of unit cubes is the product of three consecutive numbers which always contains the factors 2 and 3, making it a multiple of 6. The slab on the right stacks neatly on the top giving (n-1)(n)(n+1).

